Equilibrium and Torque
Equilibrium

An object is in “Equilibrium” when:

1. **There is no net force acting on the object**
2. **There is no net Torque** *(we’ll get to this later)*

In other words, the object is **NOT experiencing linear acceleration** or rotational acceleration.

\[
\begin{align*}
a &= \frac{\Delta v}{\Delta t} = 0 \\
\theta &= \frac{\Delta \theta}{\Delta t} = 0 \quad \text{We’ll get to this later}
\end{align*}
\]
Static Equilibrium

An object is in “Static Equilibrium” when it is NOT MOVING.

\[ v = \frac{\Box x}{\Box t} = 0 \]

\[ a = \frac{\Box v}{\Box t} = 0 \]
Dynamic Equilibrium

An object is in “Dynamic Equilibrium” when it is **MOVING with constant linear velocity** and/or rotating with constant angular velocity.

\[
a = \frac{\Delta v}{\Delta t} = 0
\]

\[
\omega = \frac{\Delta \omega}{\Delta t} = 0
\]
Equilibrium

Let’s focus on condition 1: net force = 0

\[ \vec{F} = 0 \]

The x components of force cancel

\[ \vec{F}_x = 0 \]

The y components of force cancel

\[ \vec{F}_y = 0 \]
Condition 1: No net Force

We have already looked at situations where the net force = zero. Determine the magnitude of the forces acting on each of the 2 kg masses at rest below.
Condition 1: No net Force

\[ \sum F_x = 0 \quad \text{and} \quad \sum F_y = 0 \]

\[ mg = 20 \text{ N} \]

\[ \sum F_y = 0 \]
\[ N - mg = 0 \]
\[ N = mg = 20 \text{ N} \]
Condition 1: No net Force

\[ \sum F_x = 0 \quad \text{and} \quad \sum F_y = 0 \]

\[ \sum F_y = 0 \]

\[ T_2 - mg = 0 \]

\[ T_1 = T_2 = T \]

\[ T + T = mg \]

\[ 2T = 20 \, \text{N} \]

\[ T = 10 \, \text{N} \]
Condition 1: No net Force

\[ N = mg \cos 60 \]
\[ N = 10 \text{ N} \]

\[ F_x - f = 0 \]
\[ f = F_x = mg \sin 60 \]
\[ f = 17.4 \text{ N} \]
**Condition 1: No net Force**

\[ \sum F_x = 0 \quad \text{and} \quad \sum F_y = 0 \]

![Diagram showing forces T1 and T2 with angles and mg](image)

- \( \Sigma F_x = 0 \)
- \( T2x - T1x = 0 \)
- \( T1_x = T2_x \)

Equal angles \( \Rightarrow T1 = T2 \)

- \( \Sigma F_y = 0 \)
- \( T1_y + T2_y - mg = 0 \)
- \( 2T_y = mg = 20 \text{ N} \)

- \( T_y/T = \sin 30 \)
- \( T = T_y/\sin 30 \)
- \( T = (10 \text{ N})/\sin 30 \)

**Note:** unequal angles \( \Rightarrow T1 \neq T2 \)
**Condition 1: No net Force**

\[ \sum F_x = 0 \quad \text{and} \quad \sum F_y = 0 \]

**Note:**
The y-components cancel, so \( T_{1y} \) and \( T_{2y} \) both equal 10 N.
**Condition 1: No net Force**

\[ \sum F_x = 0 \quad \text{and} \quad \sum F_y = 0 \]

\[ T_1 = 40 \text{ N} \]

\[ T_2 = 35 \text{ N} \]

\[ \sum F_y = 0 \]
\[ T_1_y - mg = 0 \]
\[ T_1_y = mg = 20 \text{ N} \]
\[ \frac{T_1_y}{T_1} = \sin 30 \]
\[ T_1 = T_y/\sin 30 = 40 \text{ N} \]

\[ \sum F_x = 0 \]
\[ T_2 - T_1_x = 0 \]
\[ T_2 = T_1 \cos 30 \]
\[ T_2 = (40 \text{ N}) \cos 30 \]

\[ T_2 = 35 \text{ N} \]
Condition 1: No net Force

\[ \Sigma F_x = 0 \quad \text{and} \quad \Sigma F_y = 0 \]

Note:
The x-components cancel
The y-components cancel
Condition 1: No net Force

*A Harder Problem!*

a. Which string has the greater tension?
b. What is the tension in each string?
a. Which string has the greater tension?

\[ \Sigma F_x = 0 \text{ so } T_{1x} = T_{2x} \]

T1 must be greater in order to have the same x-component as T2.
What is the tension in each string?

\[ \Sigma F_x = 0 \]
\[ T_2x - T_1x = 0 \]
\[ T_1x = T_2x \]
\[ T_1 \cos 60 = T_2 \cos 30 \]

\[ \Sigma F_y = 0 \]
\[ T_{1y} + T_{2y} - mg = 0 \]
\[ T_1 \sin 60 + T_2 \sin 30 - mg = 0 \]
\[ T_1 \sin 60 + T_2 \sin 30 = 20 \text{ N} \]

Note: unequal angles \( \Rightarrow T_1 \neq T_2 \)
Equilibrium

An object is in “Equilibrium” when:

1. There is no net force acting on the object
2. There is no net Torque

In other words, the object is NOT experiencing linear acceleration or rotational acceleration.

\[ a = \frac{\Delta v}{\Delta t} = 0 \]
\[ \tau = \frac{\Delta \theta}{\Delta t} = 0 \]
What is Torque?

Torque is like “twisting force”

The more torque you apply to a wheel the more quickly its rate of spin changes
Math Review:

1. Definition of angle in “radians”

\[
\theta = \frac{s}{r}
\]

\[
\theta = \frac{\text{arc length}}{\text{radius}}
\]

2. One revolution = 360° = 2\(\pi\) radians

ex: \(\pi\) radians = 180°

ex: \(\pi/2\) radians = 90°
Linear vs. Rotational Motion

- Linear Definitions
  \[ \Delta x \]
  \[ \bar{v} = \frac{\Delta x}{\Delta t} \]
  \[ \bar{a} = \frac{\Delta \bar{v}}{\Delta t} \]

- Rotational Definitions
  \[ \theta \] in radians
  \[ \omega = \frac{\Delta \theta}{\Delta t} \] in radians/sec or rev/min
  \[ \Omega = \frac{\Delta \omega}{\Delta t} \] in radians/sec/sec
**Linear vs. Rotational Velocity**

- A car drives 400 m in 20 seconds:
  a. Find the avg **linear** velocity

  \[
  \bar{v} = \frac{\Delta x}{\Delta t} = \frac{400\text{m}}{20\text{s}} = 20\text{m/s}
  \]

- A wheel spins thru an angle of 400\(\pi\) radians in 20 seconds:
  a. Find the avg **angular** velocity

  \[
  \omega = \frac{\Delta \theta}{\Delta t} = \frac{400\text{rad}}{20\text{s}} = 20\text{rad/sec}
  \]

  \[
  = 10\text{rev/sec}
  \]

  \[
  = 600\text{rev/min}
  \]
Linear vs. Rotational

Net Force $\Rightarrow$ linear acceleration
The linear velocity changes

Net Torque $\Rightarrow$ angular acceleration
The angular velocity changes
(the rate of spin changes)

$\alpha = F_{net}$
Torque

Torque is like “twisting force”

The more torque you apply to a wheel, the more quickly its rate of spin changes

Torque = Frsin\(\varnothing\)
Torque is like “twisting force”

*Imagine a bicycle wheel that can only spin about its axle.* If the force is the same in each case, which case produces a more effective “twisting force”?

This one!
Torque is like “twisting force”

Imagine a bicycle wheel that can only spin about its axle.

What affects the torque?
1. The place where the force is applied: the distance “r”
2. The strength of the force
3. The angle of the force
Torque is like “twisting force”

Imagine a bicycle wheel that can only spin about its axle.

What affects the torque?
1. The distance from the axis rotation “r” that the force is applied
2. The component of force perpendicular to the r-vector
Imagine a bicycle wheel that can only spin about its axle.

Torque = (the component of force perpendicular to \( r \))(\( r \))

Torque = \((F \sin \phi)(r)\)
Torque is like “twisting force”

Imagine a bicycle wheel that can only spin about its axle.

\[
\tau = \mathbf{F} \cdot \mathbf{r} = (F \sin \phi) (r)
\]
Cross “r” with “F” and choose any angle to plug into the equation for torque

\[ \square = (F_\perp)(r) \]
\[ \square = Fr \sin \phi \]

Since \( \square \) and \( \phi \) are supplementary angles (ie: \( \square + \phi = 180^\circ \))
\[ \sin \square = \sin \phi \]
Two different ways of looking at torque

\[ \text{Torque} = (F \sin \phi)(r) \]
\[ \text{Torque} = (F \hat{r})(r) \]

\[ F \hat{r} = F \sin \phi \]

\[ \text{Torque} = (F)(rsin\phi) \]
\[ \text{Torque} = (F)(r \hat{r}) \]

\[ F \hat{r} = F \sin \phi \]

\[ r \hat{r} \]
Imagine a bicycle wheel that can only spin about its axle.

\[ \text{Torque} = (F)(rsin\phi) \]

\( r \) is called the "moment arm" or "moment"
Equilibrium

An object is in “Equilibrium” when:

1. There is no net force acting on the object
2. There is no net Torque

In other words, the object is NOT experiencing linear acceleration or rotational acceleration.

\[ a = \frac{v}{t} = 0 \]
\[ \theta = \frac{\theta}{t} = 0 \]
Condition 2: net torque = 0

Torque that makes a wheel want to rotate clockwise is +
Torque that makes a wheel want to rotate counterclockwise is -

Positive Torque  Negative Torque
Condition 2: No net Torque

Weights are attached to 8 meter long levers at rest. Determine the unknown weights below

![Diagram of levers with weights and question marks]

- 20 N
- ??

- 20 N
- ??

- 20 N
- ??
Condition 2: No net Torque

Weights are attached to an 8 meter long lever at rest. Determine the unknown weight below

\[ \text{20 N} \quad \text{??} \]
Condition 2: No net Torque

\[ \sum T' \text{'s} = 0 \]

T2 - T1 = 0
T2 = T1

\[ F_2 r_2 \sin \varphi_2 = F_1 r_1 \sin \varphi_1 \]
\[ (F_2)(4)(\sin 90) = (20)(4)(\sin 90) \]

\[ F_2 = 20 \text{ N} \ldots \text{same as } F_1 \]
Condition 2: No net Torque
Condition 2: No net Torque

Weights are attached to an 8 meter long lever at rest. Determine the unknown weight below.

20 N

??
Condition 2: No net Torque

\[ \sum T' s = 0 \]

\[ T_2 - T_1 = 0 \]
\[ T_2 = T_1 \]
\[ F_2 r_2 \sin \phi_2 = F_1 r_1 \sin \phi_1 \]
\[ (F_2)(2)(\sin 90) = (20)(4)(\sin 90) \]

\[ F_2 = 40 \text{ N} \]

(force at the fulcrum is not shown)
Condition 2: No net Torque

20 N

40 N
Condition 2: No net Torque

Weights are attached to an 8 meter long lever \textbf{at rest}. Determine the unknown \textbf{weight} below
**Condition 2: No net Torque**

\[ T_2 - T_1 = 0 \]
\[ T_2 = T_1 \]

\[ F_2 r_2 \sin \phi_2 = F_1 r_1 \sin \phi_1 \]
\[ (F_2)(2)(\sin 90) = (20)(3)(\sin 90) \]

\[ F_2 = 30 \text{ N} \]

*force at the fulcrum is not shown*
Condition 2: No net Torque

20 N

30 N
In this **special case** where
- the pivot point is in the middle of the lever,
- and $\phi_1 = \phi_2$
$F_1R_1\sin\phi_1 = F_2R_2\sin\phi_2$
$F_1R_1 = F_2R_2$

\[
\begin{align*}
(20)(4) &= (20)(4) \\
(20)(4) &= (40)(2) \\
(20)(3) &= (30)(2)
\end{align*}
\]
More interesting problems
(*the pivot is not at the center of mass*)

Masses are attached to an 8 meter long lever **at rest**. The lever has a mass of 10 kg. Determine the unknown **weight** below.

![Diagram of lever with masses and forces]
Trick: gravity applies a torque “equivalent to”
(the weight of the lever)($R_{cm}$)
$T_{cm} = (mg)(r_{cm}) = (100 \text{ N})(2 \text{ m}) = 200 \text{ Nm}$

More interesting problems
*(the pivot is not at the center of mass)*

Masses are attached to an 8 meter long lever at rest. The lever has a mass of 10 kg.
Masses are attached to an 8 meter long lever at rest.
The lever has a mass of 10 kg.
Determine the unknown weight below.

\[ \Sigma T's = 0 \]

\[ T2 - T1 - Tcm = 0 \]
\[ T2 = T1 + Tcm \]

\[ F_2 r_2 \sin \phi_2 = F_1 r_1 \sin \phi_1 + F_{cm} R_{cm} \sin \phi_{cm} \]

\[ (F_2)(2)(\sin 90) = (20)(6)(\sin 90) + (100)(2)(\sin 90) \]

\[ F_2 = 160 \text{ N} \]

(force at the fulcrum is not shown)
Other problems:

Sign on a wall#1 (massless rod)
Sign on a wall#2 (rod with mass)
Diving board (find ALL forces on the board)
Push ups (find force on hands and feet)
Sign on a wall, again
Sign on a wall #1

A 20 kg sign hangs from a 2 meter long massless rod supported by a cable at an angle of 30° as shown. Determine the tension in the cable.

We don’t need to use torque if the rod is “massless”!

\[ Ty = mg = 200N \]

\[ \frac{Ty}{T} = \sin30 \]

\[ T = \frac{Ty}{\sin30} = 400N \]
Sign on a wall #2

A 20 kg sign hangs from a 2 meter long rod that has a mass of 10 kg and is supported by a cable at an angle of 30° as shown. Determine the tension in the cable “$F_T$”.

(force at the pivot point is not shown)
Sign on a wall #2

A 20 kg sign hangs from a 2 meter long rod that has a mass of 10 kg and is supported by a cable at an angle of 30° as shown. Determine the tension in the cable.

\[ \sum T = 0 \]
\[ T_{FT} = T_{cm} + T_{mg} \]
\[ F_T(2)\sin30 = 100(1)\sin90 + (200)(2)\sin90 \]
\[ F_T = 500 \text{ N} \]

(force at the pivot point is not shown)
Diving board

A 4 meter long diving board with a mass of 40 kg.

a. Determine the downward force of the bolt.
b. Determine the upward force applied by the fulcrum.
Diving board

A 4 meter long diving board with a mass of 40 kg.

a. **Determine the downward force of the bolt.**

*(Balance Torques)*

\[ \sum T = 0 \]

bolt

\[ R_1 = 1 \quad R_{cm} = 1 \]

\[ F_{bolt} = 400 \text{ N} \quad F_{cm} = 400 \text{ N} \]

*(force at the fulcrum is not shown)*
Diving board

A 4 meter long diving board with a mass of 40 kg.

a. Determine the downward force of the bolt.
   (Balance Torques)

b. Determine the upward force applied by the fulcrum.
   (Balance Forces)

\[ F = 800 \text{ N} \quad \sum F = 0 \]

\[ F_{\text{bolt}} = 400 \text{ N} \quad F_{\text{cm}} = 400 \text{ N} \]
Remember:

An object is in “Equilibrium” when:

a. There is no net Torque
   \[ \nabla/ = 0 \]

b. There is no net force acting on the object
   \[ \nabla F = 0 \]
Push-ups #1

A 100 kg man does push-ups as shown

Find the force on his hands and his feet

Answer:

\[ F_{\text{hands}} = 667 \text{ N} \]
\[ F_{\text{feet}} = 333 \text{ N} \]
A 100 kg man does push-ups as shown.

Find the force on his hands and his feet.

\[
\sum T = 0 \\
T_H = T_{cm} \\
F_H(1.5)\sin60 = 1000(1)\sin60
\]

\[
F_H = 667 \text{ N}
\]

\[
\sum F = 0 \\
F_{feet} + F_{hands} = mg = 1000 \text{ N}
\]

\[
F_{feet} = 1000 \text{ N} - F_{hands} = 1000 \text{ N} - 667 \text{ N}
\]

\[
F_{feet} = 333 \text{ N}
\]
Push-ups #2

A 100 kg man does push-ups as shown

Find the force acting on his hands
Push-ups #2
A 100 kg man does push-ups as shown

(force at the feet is not shown)

Force on hands:

\[ \sum T = 0 \]
\[ T_H = T_{cm} \]
\[ F_H(1.5)\sin90 = 1000(1)\sin60 \]

\[ F_H = 577 \text{ N} \]
Sign on a wall, again

A 20 kg sign hangs from a 2 meter long rod that has a mass of 10 kg and is supported by a cable at an angle of 30° as shown.

Find the force exerted by the wall on the rod

\[ FW = ? \]

(Forces and angles NOT drawn to scale)
**Find the force exerted by the wall on the rod**

\[ FW_x = FT_x = 500N(\cos30) \]
\[ FW_x = 433N \]

\[ FW_y + FT_y = F_{cm} + mg \]
\[ FW_y = F_{cm} + mg - FT_y \]
\[ FW_y = 300N - 250 \]
\[ FW_y = 50N \]

**forces and angles NOT drawn to scale!**